

# Classification of the Horndeski cosmologies via Noether symmetries

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**Abstract** Adopting Noether point symmetries, we classify and integrate dynamical systems coming from Horndeski cosmologies. The method is particularly effective both to select the form of Horndeski models and to derive exact cosmological solutions. Starting from the Lagrangians selected by the Noether symmetries, it is possible to derive several modified theories of gravity like  $f(R)$  gravity, Brans–Dicke gravity, string inspired gravity and so on. In any case, exact solutions are found out.

## 1 Introduction

The  $\Lambda$ -Cold Dark Matter Model ( $\Lambda$ CDM) can be considered the cosmological standard model supported by the majority of the cosmological observations. Indeed, type Ia Supernovae, galaxy clustering, Cosmic Microwave Background Radiation, and other observational tests, all confirm a coherent snapshot where the Hubble fluid is dominated by a cosmic fluid that accelerates the Universe and a form of matter allowing the clustering of structures. These components constitute the so called cosmic *dark side*, i.e. dark energy and dark matter. Despite of its great success in representing today’s cosmological view of the Universe,  $\Lambda$ CDM model is plagued with several shortcomings that must be framed in a self-consistent cosmological model. Besides the difficulties to find suitable candidates for dark matter particles from direct and indirect searches, to confirm (or not) the existence of supersymmetry at TeV-scales, as well as other problems [1–3], the most significant one, is the tiny value of the cosmological constant [4,5].

The inability of general relativity (GR), together with the  $\Lambda$ CDM model, to constitute a complete theory capable of describing the gravitational interactions at all scales led the scientific community to pursue new approaches by which GR should be modified or extended at infrared and ultraviolet scales. Many of the proposed alternatives are motivated by the necessity of fitting dark sector issues. Several theories [6–10] with extra degrees of freedom propagated by scalar fields (quintessence, k-essence, kinetic braiding), as well as geometric extensions of GR, like  $f(R)$  gravity [11] or  $f(T)$  teleparallel-gravity [12], have been suggested, during the last two decades to address the observed accelerating expansion of the Universe as well as the clustering of structures [13,14]. In 1974, Horndeski developed [17] the most general scalar–tensor theory (with a single scalar field) with second order field equations.<sup>1</sup> In [18,19], the Horndeski theory has been reconsidered according to a generalization of the covariant galileon models, already proposed in [20], as the decoupling limit of the graviton in the Dvali–Gabadadze–Porrati model.

Starting from the previous approach, a lot of progress has been done and the Horndeski theory can now be considered as a general theory from which several modified theories of gravity can be recovered. Scalar–tensor models, such as Brans–Dicke, k-essence, kinetic braiding, as well as the scalar–tensor analogue of  $f(R)$  gravity, are nothing else but special cases of the Horndeski action. Apart from cosmology, significant progress has been done at smaller scales in this theory. Specifically, charged black hole solutions have been studied in the context of this theory [21–25]; numerical simulations for neutron stars in specific subclasses of this theory have also been developed [26,27]. Recently, in [28], the authors reviewed the Horndeski cosmologies that

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<sup>1</sup> Theories with higher than second order equations of motion are, in most cases, plagued by the so called Ostrogradski instability and thus give rise to ghost degrees of freedom.